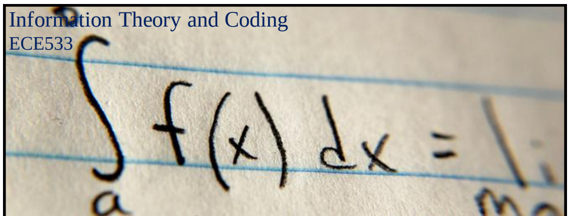
Information Theory and Coding
ECE533



# Foundation for Error Correcting

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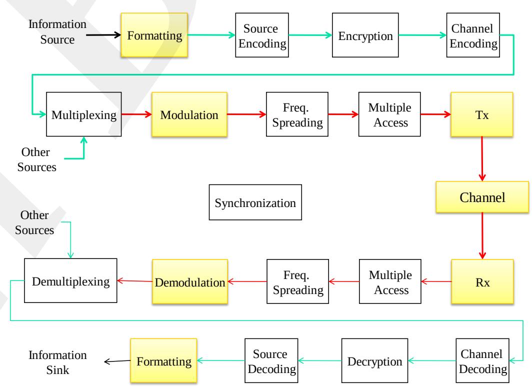
## Overview

- Modulation: BPSK, 8PSK, QPSK Constellation diagram
- Limits and Performance of Channel: BER
- Shannon and Nyquist Theorems
- AWGN and BSC Channel Models



## Communication System








## Modulation

- Analog Modulation?
- Digital Modulation?
  - ASK, PSK, FSK
  - BPSK
    - Constellation of BPSK, 8-PSK



## Digital Modulation

- ASK, FSK, PSK, QPSK, 8-PSK, MSK
  - How it is done
  - Why we need it
  - Which is better in terms of
    - BW Required
    - Performance (BER)
    - Circuit /HW/Requirement
  - M-ary?
    - Why it is required, Advantages, limitation,

## BPSK

Let $\{\ldots, b_{-2}, b_{-1}, b_0, b_1, b_2, \ldots\}$ represent a sequence of bits, $b_i \in \{0, 1\}$ every $T$ seconds.

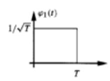
$P_1 = P(b_i = 1)$ and $P_0 = P(b_i = 0)$.

$\tilde{b}_i = (2b_i - 1)$ or $\tilde{b}_i = -(2b_i - 1)$ $\pm 1$-valued bit

$a_i = \sqrt{E_b}(2b_i - 1) = -\sqrt{E_b}(-1)^{b_i} = \sqrt{E_b}\tilde{b}_i$

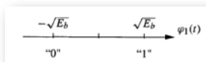
$\phi_1(t)$ is a unit-energy signal,

$$\int_{-\infty}^{\infty} \varphi_1(t)^2\, dt = 1,$$




$a_i\varphi_1(t - iT)$

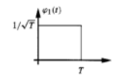
$$\int_{-\infty}^{\infty} (a_i\varphi_1(t))^2\, dt = E_b.$$



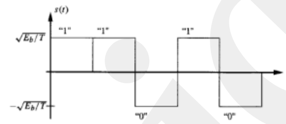




## BPSK



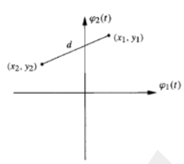





$$s(t) = \sum_i a_i\varphi_1(t - iT).$$




## 8-PSK



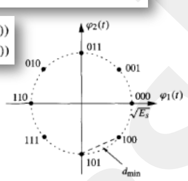

- More General

$$s(t) = x_1\varphi_1(t) + y_1\varphi_2(t).$$

$$\langle\varphi_1(t), \varphi_1(t)\rangle = 1 \quad \langle\varphi_2(t), \varphi_2(t)\rangle = 1 \quad \langle\varphi_1(t), \varphi_2(t)\rangle = 0.$$

$s_1(t) = x_1\varphi_1(t) + y_1\varphi_2(t)$ (i.e., $s_1(t) \leftrightarrow (x_1, y_1)$)

$s_2(t) = x_2\varphi_1(t) + y_2\varphi_2(t)$ (i.e., $s_2(t) \leftrightarrow (x_2, y_2)$)



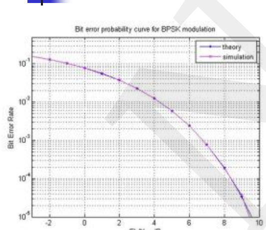


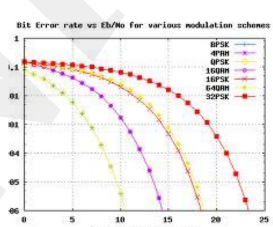


## QPSK




## BER vs SNR

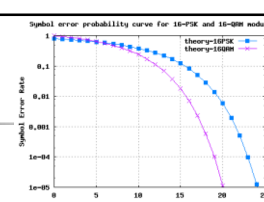


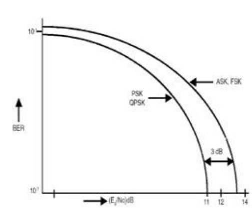



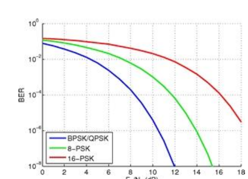




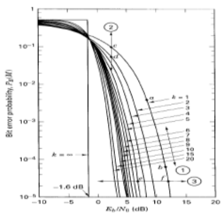
## BER vs SNR



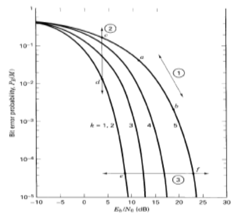



## Eo/No with M-ary signals



- Orthogonal signaling



Multiple phase signaling



## Eo/No with M-ary signals

- What is M-ary Modulation??

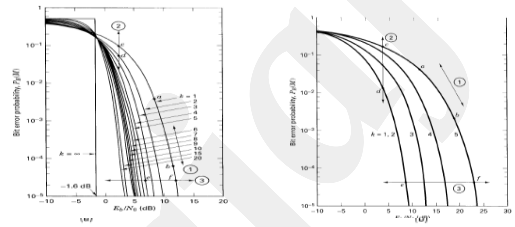


**Figure 9.1** Bit error probability versus $E_b/N_0$ for coherently detected *M*-ary signaling. (a) Orthogonal signaling. (b) Multiple phase signaling.



## BER Vs Eb/N0 & Modulation

- You should now
  - Know, How BER is computed
  - Know, How Eb/N0 effects BER
  - Able to read BER Vs Eb/N0 Graph
  - Able to make sense of each point and curve in graph
  - Able to understand which better
- Reference: TODD K MOON book



## Limits and Bounds
## Shannon's Theorem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## Goals of Comm. Engineer

- Rb ↑
- BER ↓
- Required power or Eb/No ↓
- Required BW ↓
- System utilization ↑
- System Complexity ↓



## Constrain to achieve goals

- Nyquist theoretical minimum BW requirement
- Shannon-Hartley capacity theorem- Shannon Limit
- Government Regulations (e.g. Freq. allocations)
- Technological Limitations (e.g. State-of the-art components)
- Other system requirements

## Nyquist Bandwidth

- Theoretical minimum bandwidth required for the baseband transmission of $R_s$ **symbol** per second without ISI is $R_s/2$

$$BW_{min} = \frac{R_s}{2}$$

- Example: For transmitting 2000 symbols per second minimum BW required is 1Khz
- This is Basic theoretical constrain.
- 2 symbol per second per Hz
- Practically 1.8 to 1.4 symbols/s/Hz due to non ideal filters

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## Nyquist Bandwidth

- Relation between BW and Rs

$$BW_{min} = \frac{R_s}{2}$$

- Relation between BW and Bit rate ???
  - Number of bits per sample??
  - M-ary modulation, M levels
  - k= $\log_2 M$, bits per symbol
- Bit rate R = $kR_s$

$$R_s = \frac{R}{k} = \frac{R}{\log_2 M} \qquad BW_{min} = \frac{R}{2\log_2 M}$$

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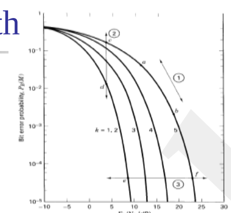
## Nyquist Bandwidth

$$BW_{min} = \frac{R}{2\log_2 M}$$



- Example: for 8-ary PSK system and 9.6kbps bit rate the minimum BW required is =?
- For 4-ary PSK
- For 16-ary PSK

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## Shannon's Capacity Theorem

- "The cannel capacity C is the function of the average received signal power S, average noise power N and bandwidth W"
  - Shannon-Hartley Capacity Theorem

$$C = W\log_2\left(1+\frac{S}{N}\right)$$

Not in dB, it is ratio

$\text{SNR (in dB)} = 10\log_{10}\frac{S}{N}$.

- Theoretically it is possible to transit the information over channel at rate R, R ≤ C with small probability of error by using complex coding.
- But at rate R>C, it is not possible.

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## Questions

- If the SNR is 20 dB, and the bandwidth available is 4 kHz, which is appropriate for telephone communications. Compute Capacity C.
- If the requirement is to transmit at 50 kbit/s, and a bandwidth of 1 MHz is used, then the minimum S/N required is ?
- Example……………….
- Let's take the example of W-CDMA (Wideband Code Division Multiple Access), the bandwidth = 5 MHz, you want to carry 12.2 kbit/s of data (AMR voice), then the required SNR is given by $2^{12.2/5000} - 1$ corresponding to an SNR of −27.7 dB for a single channel. This shows that it is possible to transmit using signals which are actually much weaker than the background noise level, as in spread-spectrum communications. However, in W-CDMA the required SNR will vary based on design calculations.

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## Shannon's Capacity Theorem

- This shows that S, N, and W limits the Transmission rate, not the error probability

$$N = NoW$$

- Normalized channel capacity
- $\frac{C}{W} = \log_2\left(1+\frac{S}{NoW}\right)$ bits/s/Hz
- Normalized channel bandwidth
- $\frac{W}{C} = -\log_2\left(1+\frac{S}{NoW}\right)$ Hz/bits/s

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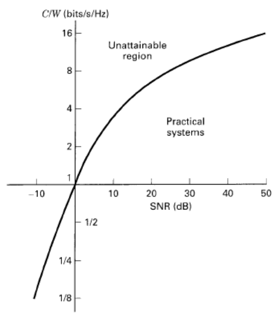
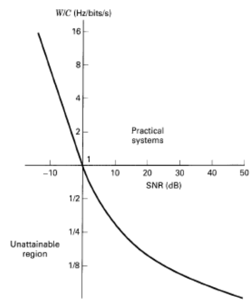
## Graphical representation

$$\frac{C}{W} = \log_2\left(1 + \frac{S}{NoW}\right)$$

$$\frac{W}{C} = -\log_2\left(1 + \frac{S}{NoW}\right)$$




## Shannon's Capacity Theorem

- If R=C

$$\frac{S}{NoC} = \frac{E_b}{No} \qquad \frac{S}{No} = \frac{E_b R}{No}$$

$$\frac{C}{W} = \log_2\left(1 + \frac{E_b}{No}\left(\frac{C}{W}\right)\right)$$

$$2^{C/W} = 1 + \frac{E_b}{No}\left(\frac{C}{W}\right)$$

$$\frac{E_b}{No} = \frac{W}{C}\left(2^{C/W} - 1\right)$$




## Shannon's Limit

$$\frac{C}{W} = \log_2\left(1 + \frac{E_b}{No}\left(\frac{C}{W}\right)\right)$$

- There is limiting value of Eb/No, below which there can be NO Error-Free communication at any information rate
- $\lim_{x \to 0} (1 + x)^{1/x} = e$ $\qquad x = \frac{E_b}{N_0}\left(\frac{C}{W}\right)$

$$\frac{C}{W} = x \log_2 (1 + x)^{1/x}$$

In the limit, as $C/W \to 0$, we get

$$\frac{E_b}{N_0} = \frac{1}{\log_2 e} = 0.693$$

$$1 = \frac{E_b}{N_0} \log_2 (1 + x)^{1/x}$$

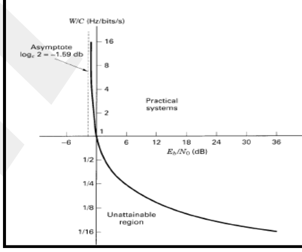
or, in decibels,

$$\frac{E_b}{N_0} = -1.6 \text{ dB}$$




## Shannon Limit

- Shannon's limit (Value of Eb/No)



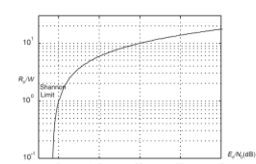
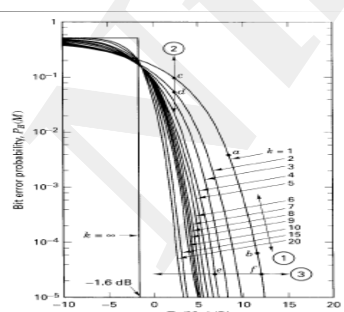




Fig. 2.7 The bandwidth efficiency diagram.



## Shannon's Limit







## Noisy Channel Coding Theorem

  - Information rate H/Ts
  - Capacity C bits per use of channel
- Noisy Channel Coding Theorem
- 1 $\frac{H(X)}{T_s} \le \frac{C}{T_c}$
- 2 $\frac{H(X)}{T_s} > \frac{C}{T_c}$ $\qquad$ C/T—Critical Rate

## Summary

- Nyquist limit

$$BW_{\min} = \frac{R}{2\log_2 M}$$

- Shannon's theorem

$$\frac{C}{W} = \log_2\left(1 + \frac{S}{NoW}\right)$$

- Shannon's Limit

$$\frac{E_b}{N_0} = -1.6\text{ dB}$$

- Noisy Channel Theorem

$$\frac{H(X)}{T_s} \le \frac{C}{T_c}$$

$$\frac{H(X)}{T_s} > \frac{C}{T_c}$$

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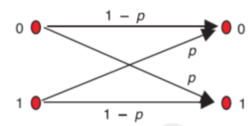
## Binary Symmetric Channel-BSC




$$P(0|1) = P(1|0) = p$$

$$P(1|1) = P(0|0) = 1 - p$$

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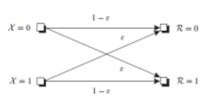
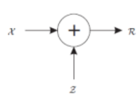
**Binary symmetric channel**




- Bit error probability $\varepsilon$
- Channel capacity

$$C = 1 + \varepsilon \log_2(\varepsilon) + (1 - \varepsilon)\log_2(1 - \varepsilon)$$

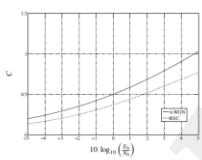
**AWGN channel**




- Signal-to-noise ratio $\frac{S}{N}$
- Channel capacity

$$C = \frac{1}{2}\log_2\left(1 + \frac{S}{N}\right)$$

**Channel capacity of BSC vs AWGN channel**




- Signal-to-noise ratio of AWGN channel

$$\frac{S}{N} = \frac{E_b}{N_0/2}$$

- Bit error probability of binary symmetric channel

$$\varepsilon = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

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