

Signal & Systems
ECE220

Systems

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Overview

- Objective of Subject
- Syllabus: overview
- Reference books, Papers, Journals
- Introduction to subject
- Prerequisite Knowledge
- Review of Basics
- Summary

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Systems

- What is systems?
- Examples

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Introduction

The intent of this introduction is to give the reader an idea about **Signals and Systems** as a field of study and its applications. But we must first, at least vaguely define what signals and systems are.

Signals are functions of one or more variables .
Systems respond to an input signal by producing an output signal.

Examples of signals include :

- A **voltage signal**: voltage across two points varying as a function of time.
- A **force pattern**: force varying as a function of 2-dimensional space.
- A **photograph**: color and intensity as a function of 2-dimensional space.
- A **video signal**: color and intensity as a function of 2-dimensional space and time.

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Systems

- Examples of systems include :
 - An **oscilloscope**: takes in a voltage signal, outputs a 2-dimensional image characteristic of the voltage signal.
 - A **computer monitor**: inputs voltage pulses from the CPU and outputs a time varying display.
 - An **accelerating mass** : force as a function of time may be looked at as the input signal, and velocity as a function of time as the output signal.
 - A **capacitance**: terminal voltage signal may be looked at as the input, current signal as the output.

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Examples of mechanical and electrical systems

- You are surely familiar with many of these signals and systems and have probably analyzed them as well, but in isolation. For instance, you must have studied accelerating masses in a mechanics course (see Fig (a)), and capacitances in an electrostatic course (see Fig (b)), separately

- As you can see, there is a similarity in the way the input signal is related to the output signal. These similarities will interest us in this course as we may be able to make inferences common to both these systems from these similarities.

We will develop very general tools and techniques of analyzing systems, independent of the actual context of their use. Our approach in this course would be to define certain properties of signals and systems (inspired of course by properties real-life examples we have), and then link these properties to consequences. These "links" can then be used directly in connection with a large variety of systems: electrical, mechanical, chemical, biological... knowing only how the input and output signal are related! Thus, our focus when dealing with signals and systems will be on the relationship between the input and output signal and not really on the internals of the system.

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#Refer to Chapter 1 of Oppenheim

Systems

- System
 - Properties of System
 - With memory or Without Memory
 - Linear or Nonlinear
 - Time invariant or Time variant
 - Stable or Unstable
 - Causal or Non-causal
 - Invertible or Non invertible

LTI Sys

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With Memory and Without Memory

- System, whose output depend on past or/and future input, is called with memory system
 - Also called as dynamic system
 - $y(t) = x(t-1) + x(t+1)$
- System, whose output depend only on present input is called without memory system
 - Also called as Static System
 - $y(t) = 2x(t)$

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Linear and Nonlinear

- Linear very interesting class of systems
- System that satisfies superposition theorem are called as Linear system else they are non linear
- Superposition Theorem:
 - Additively property
 - Homogeneity property

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#Ref: Lathi

Linear System

- A linear system exhibits the **additivity** property:

$x_1 \rightarrow y_1$
 $x_2 \rightarrow y_2$
 $x_1 + x_2 \rightarrow y_1 + y_2$
- It also must satisfy the **homogeneity** or **scaling** property:

$x \rightarrow y$
 $kx \rightarrow ky$
- These can be combined into the property of **superposition**:

$x_1 \rightarrow y_1$
 $x_2 \rightarrow y_2$
 $k_1x_1 + k_2x_2 \rightarrow k_1y_1 + k_2y_2$
- A non-linear system is one that is NOT linear (i.e. does not obey the principle of superposition)

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Time invariant and Time varying

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Time Invariant

- Time-invariant system is one whose parameters do not change with time:

$$\begin{array}{c}
 x(t) \rightarrow \text{TI System} \rightarrow y(t) \xrightarrow{\text{delay by T seconds}} y(t-T) \\
 x(t) \xrightarrow{\text{delay by T seconds}} x(t-T) \rightarrow \text{TI System} \rightarrow y(t-T)
 \end{array}$$

- Linear time-invariant (LTI) systems – main concern for this course and the Control course in 2nd year. (Lathi: LTIC = LTI continuous, LTID = LTI discrete)

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Causal and non Causal

Causal system – output at t_0 depends only on $x(t)$ for $t \leq t_0$

I.e. present output depends only on the past and present inputs, **not on future inputs**

Any practical **REAL TIME system must be causal.**

Noncausal systems are important because:

1. Realizable when the independent variable is something other than "time" (e.g. space)
2. Even for temporal systems, can prerecord the data (non-real time), mimic a non-causal system
3. Study upper bound on the performance of a causal system

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Stable and Unstable

- **BIBO Stability**

Externally stable systems: **Bounded input** results in **bounded output** (system is said to be stable in the **BIBO** sense)

Stability of a system – mostly covered on the Control course

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Invertible

- Let a system S produces $y(t)$ with input $x(t)$, if there exists another system S_1 , which produces $x(t)$ from $y(t)$, then S is invertible
- Essential that there is **one-to-one mapping** between input and output
- For example if S is an amplifier with gain G, it is invertible and S_1 is an attenuator with gain 1/G
- Apply S_1 following S gives an **identity system** (i.e. input $x(t)$ is not changed)

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Systems with Signal

- **Convolution**

Convolution Integral:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

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Example 1

Determine graphically $y(t) = x(t) * h(t)$ for $x(t) = e^{-t}u(t)$ and $h(t) = e^{-2t}u(t)$.

$$y(t) = \int_0^t x(\tau)h(t - \tau) d\tau \quad t \geq 0$$

Remember: variable of integration is τ , not t

L2.4-1 p183

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Example 1

$$y(t) = \int_0^t e^{-\tau} e^{-2(t-\tau)} d\tau$$

$$= e^{-2t} \int_0^t e^{\tau} d\tau$$

$$= e^{-t} - e^{-2t}$$

Moreover, $y(t) = 0$ for $t < 0$, so that

$$y(t) = (e^{-t} - e^{-2t})u(t)$$

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Connections of systems

- Parallel connected system

$$y(t) = h_1(t) * x(t) + h_2(t) * x(t)$$

- Cascade systems & Commutative property

$$y(t) = [h_1(t) * h_2(t)] * x(t)$$

L2.4-3 p192

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Convolution table

- Use table to find convolution results easily:

| No. | $x_1(t)$ | $x_2(t)$ | $x_1(t) * x_2(t) = x_2(t) * x_1(t)$ |
|-----|------------------------|------------------------|---|
| 1 | $x(t)$ | $\delta(t - T)$ | $x(t - T)$ |
| 2 | $e^{\lambda t} u(t)$ | $u(t)$ | $\frac{1 - e^{\lambda t}}{-\lambda} u(t)$ |
| 3 | $u(t)$ | $u(t)$ | $t u(t)$ |
| 4 | $e^{\lambda_1 t} u(t)$ | $e^{\lambda_2 t} u(t)$ | $\frac{e^{\lambda_1 t} - e^{\lambda_2 t}}{\lambda_1 - \lambda_2} u(t)$ $\lambda_1 \neq \lambda_2$ |
| 5 | $e^{\lambda t} u(t)$ | $e^{\lambda t} u(t)$ | $t e^{\lambda t} u(t)$ |
| 6 | $t e^{\lambda t} u(t)$ | $e^{\lambda t} u(t)$ | $\frac{1}{2} t^2 e^{\lambda t} u(t)$ |

L2.4 p17

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